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Innovative Optimal Control Methodology of Heat Dissipation in Electronic Devices

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As the size of electronic devices fabricated using complementary metal-oxide semiconductor technology decreases with increased operating frequency, the chip energy density increases, resulting in higher temperatures, reduced device lifetime, and reliability issues. Precisely managing a time-varying electronic package heat source in real time has become crucial to designing an excellent power dissipation mechanism. This work is the first to develop an innovative algorithm that combines the linear quadratic Gaussian regulator and the input estimation method for optimizing heat-dissipation control. The main purpose of the input estimation method, which is a combination of a Kalman filter and the recursive least-squares estimator, is to estimate in real time the unknown time-varying heat flux by simulating the electronic package surface temperature. Furthermore, the linear quadratic Gaussian regulator is adopted to analyze the feedback gain to control the heat dissipation. Comparing the linear quadratic Gaussian/input estimation method and linear quadratic Gaussian regulator with three typical time-varying heat-flux models indicates that the proposed algorithm offers superior performance in terms of short time response and excellent heat-dissipation control via accurate tracking of the heat flux generated by electronic devices. The simulation results reveal that an effective and optimal heat-dissipation controller can be implemented for the cooling system using the proposed algorithm.

Nomenclature

B	=	sensitivity matrix defined by Eq. (16), $m^2 {}^\circ C/W$
C_p	=	specific heat of package seal, $J/(kg^\circ C)$
$E(\bullet)$	=	expected value
G	=	coefficient vector defined by Eq. (2a), $m^2 {}^\circ C/J$
H	=	the measurement matrix defined by Eq. (8)
h	=	convection heat transfer coefficient, $W/(m^2 {}^\circ C)$
I	=	identity matrix
J_i	=	quadratic performance index defined by Eq. (24)
K	=	Kalman gain
K_b	=	Kalman gain of input estimation, $W/(m^2 {}^\circ C)$
K_r	=	regulator gain, $W/(m^2 {}^\circ C)$
K_s	=	thermal conductivity, $W/(m^\circ C)$
k, j	=	time (discrete), s
k_f	=	time steps for $t = t_f$, s
L	=	thickness of package material, m
M	=	sensitivity matrix defined by Eq. (17)
N	=	total number of spatial nodes for $x = L$
n	=	number of spatial nodes for $0 \leq x < L$
P	=	filter's error-covariance matrix defined by Eq. (10), ${}^\circ C^2$
P_b	=	error-covariance matrix defined by Eq. (19), $(W/m^2)^2$
P_1	=	matrix defined by Eq. (29), ${}^\circ C^{-2}$
Q	=	process noise covariance matrix, $(W/m^2)^2$
Q_c	=	weights of the control, m^4/W^2
Q_0	=	weighting matrix of state variable error for final time, ${}^\circ C^{-2}$
Q_s	=	weights of the state variable error, ${}^\circ C^{-2}$
Q_1	=	weighting matrix of state variable error for overall process, ${}^\circ C^{-2}$
q	=	heat flux, W/m^2
R	=	measurement noise covariance, ${}^\circ C^2$
s	=	innovation covariance defined by Eq. (11), ${}^\circ C^2$
T	=	temperature, °C
t	=	time (continuous), s
t_f	=	final time, s
u	=	unknown boundary heat-flux input, W/m^2
$u_{\max}$	=	permission value of control variable, W/m^2
w	=	process noise vector, W/m^2
X	=	state vector, °C
X_d	=	design working temperature, °C
$\bar{X}$	=	estimation of state for $q = 0$, °C
x	=	axial coordinate, m
x_m	=	location of thermocouple, m
z	=	observation state vector, °C
$\bar{z}$	=	bias innovation, °C
α	=	thermal diffusivity, m^2/s
Γ	=	coefficient matrices defined by Eq. (7), $m^2 {}^\circ C/W$
γ	=	memory factor, defined by Eq. (18)
Δt	=	sampling time interval, s
Δx	=	spatial interval, m
δ	=	Dirac delta function
$\varepsilon_{i,\max}$	=	maximum tolerance of state variable error, °C
η	=	random variable
Λ	=	coefficient matrix defined by Eq. (6), $m^2 {}^\circ C/W$
ν	=	measurement noise vector, °C
ρ	=	density of package seal, kg/m^3
σ	=	standard deviation, °C
τ	=	time, s
Φ	=	state transition matrix defined by Eq. (5)
χ	=	state variable error, °C
Ψ	=	coefficient matrix defined by Eq. (2c), s^{-1}
Ω	=	coefficient vector defined by Eq. (2e), $m^2 {}^\circ C/J$

Subscripts

0	=	initial temperature
*	=	extensible controller
air	=	natural convection
exact	=	exact heat flux
$1, 2, \ldots, n$	=	number of spatial nodes
∞	=	ambient temperature

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Superscripts

^ = estimated
T = transpose of matrix

Introduction

AS TECHNOLOGICAL advances continue to be made, electronic products are being developed that are lightweight and operate at higher power density and higher performance. Semiconductor components are becoming smaller because manufacturing processes are improving. The number of transistors per chip unit area has therefore increased. Therefore, the heat produced per active component area, such as chipsets, memory modules, and image processors, has markedly increased. Controlling the temperature of these components to avoid heat damage has become an important issue. The heat produced during high-speed computer operation increases with the CPU calculation speed [1]. The lifetime and reliability of any electronic component falls as the operating temperature increases [2,3]. Consequently, effectively dealing with the heat-dissipation issue is the best way to increase the life span of electronic products. Predicting heat production is the key to enhancing the reliability of the entire operating system.

In the past decades, there have been many studies on controlling the rise of temperature in electronic devices [4–7]. These studies focused on reducing the interfacial thermal impedance, which may be called passive thermal control. In the meantime, some active thermal control methods have been proposed to lift the limitations in passive thermal control methods [8–11]. Some are feedback control systems [8,9] and others are not [10,11]. However, there are two difficult areas in the application of feedback control systems to heat dissipation in electronic devices. One is the thermal time delay problem and the other is the short period of time for control action. The main cause of these difficulties is poor prediction of the time-varying heat generated by the CPU. Thus, accurate real-time estimation of the time-varying heat generated by the CPU plays an important role in these active thermal control methods.

In the past, much research on the optimal control design problem was focused on well-defined systems. This means that all system parameters are given initially and the system input is also well known. Under these conditions, the linear quadratic Gaussian (LQG) regulator [12] could easily solve the optimization problem with good accuracy. However, it is difficult to obtain a good solution for heat-dissipation control design if thermal input or other parameters are deterministic but uncertain. For instance, the heat generated by a CPU may be defined as the thermal input, and thermal conductivity may be defined as a thermal property. Sometimes, these values are unknown and cannot be measured. Accordingly, the approach proposed here involves a combined control algorithm that could concurrently estimate unknown parameters or thermal input online, using the input estimation (IE) method, and solve the optimal control problem based on the LQG regulator.

In 1996, Tuan et al. [13] proposed the online IE method to estimate the unknown heat flux in a linear inverse heat conduction problem. The IE method is composed of the Kalman filter technique and the recursive least-squares estimator. The Kalman filter is used to generate the residual innovation sequence, which contains the bias or systematic errors caused by the implicit unknown time-varying input and also contains the variance or random noise caused by the measurement errors. Based on the regression model, the recursive least-squares estimator is proposed to estimate the magnitude of the unknown heat flux generated by an electronic device. The linear inverse heat conduction problem has been fully addressed by Tuan et al. [14] and Ji et al.[15]. However, it was not extended to solve the optimal thermal control problem. Consequently, this work studies the optimal thermal control problem of a linear heat-conducting system under the influence of unknown thermal input. The main objective of this work is to develop an adaptive control algorithm that predicts the optimal value of time-varying heat dissipation, which must be removed by the cooling system, with the problem corrupted by modeling noise and unknown thermal input. After estimating the amount of heat which must be dissipated, then we can design an effective cooling system to assure that the CPU will be operated in a safe temperature regime.

In this work, a heat-dissipation model for electronic devices is defined first, and an innovative controller that combines the LQG regulator and IE method is then developed to control the electronic device heat dissipation. Elaborate experiments are simulated for three cases, the results compared and conclusions drawn.

Problem Formulation

An example is considered in the formulation of this combined theorem for application to heat-dissipating control problems. The problem being considered is the conduction of heat through a package seal to heat-dissipation components during the operation of an integrated circuit. Figure 1 details a surface-mounted package of an integrated circuit, in which a bare semiconductor die is bounded on a paddle using adhesives, attached to a lead frame with wire bounds, sealed with a solder mask, and then mounted on a printed circuit board. Heat-spreading material is uniformly applied between the package seal and heat sink on top of the package seal. The heat sink is mounted directly on the seal to improve the heat-dissipation efficiency.

To simplify the analysis, a one-dimensional heat conduction model is employed, as outlined in Fig. 2. Consider a package seal that is a uniform thermal conductor with thickness L. An unknown heat flux $q(t)$, which is generated by the semiconductor die, acts on the package seal at $x = 0$. The other side of the package seal, where $x = L$, has a natural convection or forced convection boundary condition. However, that boundary heat flux (i.e., the control input) is unknown because the characteristics of the heat-dissipating components are unknown. A thermocouple is placed at $x = L$ to measure the temperature $z(t)$.

The one-dimensional heat conduction equation and boundary conditions are as follows:

$$K_s \frac{\partial^2 T(x,t)}{\partial x^2} = \rho C_p \frac{\partial T(x,t)}{\partial t} \qquad 0 \le x \le L, \qquad t > 0 \qquad \text{(1a)}$$

$$-K_s \frac{\partial T(x,t)}{\partial x} = q(t) \qquad x = 0, \qquad t > 0 \qquad \text{(1b)}$$

$$-K_s \frac{\partial T(x,t)}{\partial x} = u(t) = h(t)[T(x,t) - T_\infty] \qquad x = L, \qquad t > 0 \qquad \text{(1c)}$$

$$T(x,t) = T_0 \qquad 0 \le x \le L, \qquad t = 0 \qquad \text{(1d)}$$

The measurement equation is

$$z(t) = T(x,t) + v(t), \qquad x = L, \qquad t \ge 0 \qquad \text{(1e)}$$

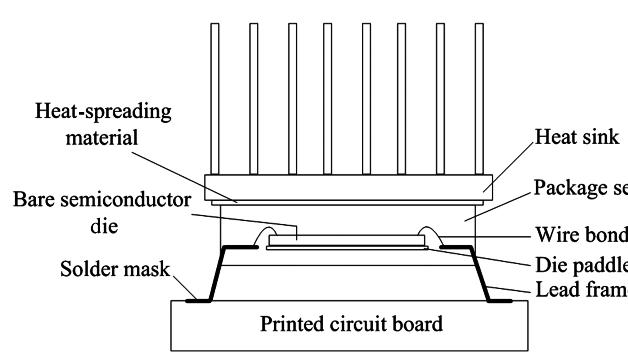

Fig. 1 Diagram of surface-mounted package of integrated circuit.

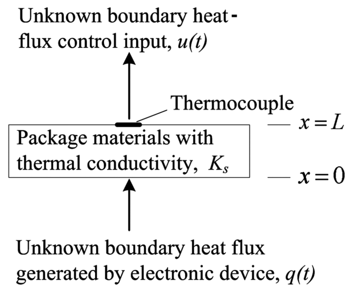

Fig. 2 Simplified heat-dissipation model for a typical electronics package.

where $\nu(t)$ is the measurement error, which is assumed to be Gaussian white noise with a zero mean.

The heat conduction equations are first transformed into state equations to use the input estimation approach and LQG regulator to solve heat-dissipation control problems. The central difference approximation [16] is used for the spatial derivative along with the boundary conditions, Eqs. (1b) and (1c), to yield the following continuous-time state equation:

$$\dot{X}(t) = \Psi X(t) + Gu(t) + \Omega[q(t) + w(t)] \tag{2a}$$

where

$$X(t) = \begin{bmatrix} T_1(t) & T_2(t) & \cdots & \cdots & T_{n-1}(t) & T_n(t) \end{bmatrix}^T \tag{2b}$$

$$\Psi = \frac{\alpha}{\Delta x^2}\begin{bmatrix} -2 & 2 & 0 & 0 & 0 & 0 \\ 1 & -2 & 1 & 0 & 0 & 0 \\ 0 & \ddots & \ddots & \ddots & 0 & 0 \\ 0 & 0 & \ddots & \ddots & \ddots & 0 \\ 0 & 0 & 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 0 & 2 & -2 \end{bmatrix} \tag{2c}$$

$$G = \frac{2\alpha}{K_s \Delta x}\begin{bmatrix} 0 \\ 0 \\ \vdots \\ \vdots \\ 0 \\ -1 \end{bmatrix} \tag{2d}$$

$$\Omega = \frac{2\alpha}{K_s \Delta x}\begin{bmatrix} 1 \\ 0 \\ \vdots \\ \vdots \\ 0 \\ 0 \end{bmatrix} \tag{2e}$$

When the continuous-time state equation, Eq. (2a), is sampled with sampling time interval Δt, the discrete-time state equation is obtained as

$$X(k+1) = \Phi X(k) + \Lambda u(k) + \Gamma[q(k) + w(k)] \tag{3}$$

where

$$X(k) = \begin{bmatrix} T_1(k) & T_2(k) & \cdots & \cdots & T_{n-1}(k) & T_n(k) \end{bmatrix}^T \tag{4}$$

$$\Phi = e^{\Psi \Delta t} \tag{5}$$

$$\Lambda = \int_{k\Delta t}^{(k+1)\Delta t} \exp[\Psi(k\Delta t - \tau)] G \, d\tau \tag{6}$$

$$\Gamma = \int_{k\Delta t}^{(k+1)\Delta t} \exp[\Psi(k\Delta t - \tau)] \Omega \, d\tau \tag{7}$$

where $w(k)$ is the process noise vector and is assumed to be Gaussian white noise with a zero mean. The covariance of the process noise is $E\{w(k)w^T(j)\} = Q\delta_{kj}$, where δ_{kj} is the Dirac delta function. The discrete-time measurement equation is

$$z(k) = HX(k) + \nu(k) \tag{8}$$

where $H = \begin{bmatrix} 0 & 0 & \cdots & \cdots & 0 & 1 \end{bmatrix}$ is the measurement matrix and $\nu(k)$ is the measurement noise vector, which is assumed to be Gaussian white noise with a zero mean. The covariance of the measurement noise is $E\{\nu(k)\nu^T(j)\} = R\delta_{kj}$.

This work develops an innovative controller using an LQG/IE algorithm that can determine the amount of heat dissipated $u(k)$ under an unknown heat flux $q(k)$ in Eq. (3).

Innovative Controller Design

Based on the assumption that the heat flux $q(k)$ is zero or a known value in Eq. (3), an optimal control result can be obtained using the LQG regulator. However, when the system being controlled has a time-varying heat-flux input $q(k)$, such as the heat flux resulting from electronic device operation, the optimal heat-dissipating control system design for the time-varying heat flux cannot easily be obtained using an LQG regulator. To resolve this issue, this work proposes a combined control theorem that uses the separation principle [12] in the calculation. The control problem is divided into two portions: parameter estimation and optimal control. The separation principle is adopted to separately address the optimal control and optimal parameter estimation problems. In solving the optimal parameter estimation problem, the heat-dissipating control quantity $u(k)$ is assumed to be a known nonrandom variable. The heat-flux input $q(k)$ is then estimated using the IE method. The parameter $q(k)$, obtained from the preceding solution, is deduced to determine a new heat-dissipating control quantity $u(k)$ in the optimal control problem solution. This recursive process is repeated until the final time is reached.

The IE method has two portions: the Kalman filter and the recursive least-squares algorithm (RLSA). The IE method uses the Kalman filter to generate the residual innovation sequence. Based on a regression model, the RLSA is proposed to extract the time-varying heat flux in real time, named as the unknown boundary heat flux generated by an electronic device. The RLSA is adopted to estimate the parameters to reduce the memory used in the calculation. This is done because only the output and the measurements from the preceding moment are required for each estimate. The detailed derivation and explanation of this technique can be found in Tuan et al. [13].

Formulation of the Kalman filter

$$\bar{X}(k/k-1) = \Phi \bar{X}(k-1/k-1) + \Lambda \hat{u}(k-1) \tag{9}$$

$$P(k/k-1) = \Phi P(k-1/k-1)\Phi^T + \Gamma Q \Gamma^T \tag{10}$$

$$s(k) = HP(k/k-1)H^T + R \tag{11}$$

$$K(k) = P(k/k-1)H^T s^{-1}(k) \tag{12}$$

$$P(k/k) = [I - K(k)H]P(k/k-1) \tag{13}$$

$$\bar{z}(k) = z(k) - H\bar{X}(k/k-1) \tag{14}$$

$$\bar{X}(k/k) = \bar{X}(k/k-1) + K(k)\bar{z}(k) \tag{15}$$

The Kalman filter is a time-varying digital filter that uses information from both the state equation, Eq. (3), and the measurement equation, Eq. (8). Embedded within the Kalman filter is a set of recursive matrix equations that permit us to perform filter analysis before any data are processed. This allows state estimation error-covariance matrix precomputation before data processing. $K(k)$ is the Kalman gain matrix, $P(k/k-1)$ is the state prediction-error-covariance matrix, $P(k/k)$ is the state filtering-error-covariance matrix, $\bar{X}(k/k-1)$ is the recursive predictor, and $\bar{X}(k/k)$ is the recursive filter.

RLSA formulation

$$B(k) = H[\Phi M(k-1) + I]\Gamma \tag{16}$$

$$M(k) = [I - K(k)H][\Phi M(k-1) + I] \tag{17}$$

$$K_b(k) = \gamma^{-1}P_b(k-1)B^T(k)[B(k)\gamma^{-1}P_b(k-1)B^T(k) + s(k)]^{-1} \tag{18}$$

$$P_b(k) = [I - K_b(k)B(k)]\gamma^{-1}P_b(k-1) \tag{19}$$

$$\hat{q}(k) = \hat{q}(k-1) + K_b(k)[\bar{z}(k) - B(k)\hat{q}(k-1)] \tag{20}$$

where $\hat{q}(k)$ is the estimated input vector, $P_b(k)$ is the error covariance of the estimated input vector, $B(k)$ and $M(k)$ are the sensitivity matrices, and $K_b(k)$ is the Kalman gain. The bias input error $\bar{z}(k)$ is caused by the measurement noise and input disturbance. In this case, the correction gain $K_b(k)$ for updating $\hat{q}(k)$ in Eq. (20) diminishes as k increases and permits $\hat{q}(k)$ to converge to the true constant value. In the time-varying case, however, we like to prevent $K_b(k)$ from reducing to zero. This is accomplished by introducing the memory factor γ. For $0 < \gamma \le 1$, $K_b(k)$ is effectively prevented from shrinking to zero. Hence, the corresponding algorithm can continuously preserve its updating ability. The γ value depends on the process noise covariance Q and the measurement noise covariance R. Usually, the R value depends on the sensor measurements. Both the Q value in the filter and the γ value in the sequential least-squares approach will interactively affect the fast adaptive capability in tracking of time-varying heat generated by the CPU. In general, if we select a large Q value, the γ value could be chosen near one and the filter memory becomes long, reducing the noise effects. For a smaller γ value, the memory becomes short and the estimation can track sudden changes occurring in the heat flux $q(k)$. For this work, we chose $\gamma = 0.875$ as a compromise between fast adaptive capability and the estimate accuracy loss.

For controller design, Eq. (3) is rewritten, substituting $q(k)$ for $\hat{q}(k)$, provided by Eq. (20). The discrete-time state equation and the measurement equation are given using

$$X(k+1) = \Phi X(k) + \Lambda u(k) + \Gamma[\hat{q}(k) + w(k)] \tag{21a}$$

$$z(k) = HX(k) + \nu(k) \tag{21b}$$

Rearrange Eq. (21a) to give

$$X(k+1) = \Phi X(k) + \Lambda[u(k) + (\Lambda^T\Lambda)^{-1}\Lambda^T\Gamma\hat{q}(k)] + \Gamma w(k) \tag{22}$$

Let the items in parentheses in Eq. (22) represent the extensible controller $u_*(k)$, and the following equation can then be obtained:

$$X(k+1) = \Phi X(k) + \Lambda u_*(k) + \Gamma w(k) \tag{23}$$

The quadratic performance index $J_i(u_*)$ is defined as

$$J_i(u_*) = E\left\{\frac{1}{2}\chi^T(k_f)Q_0\chi(k_f) + \frac{1}{2}\sum_{k=i}^{k_f-1}[\chi^T(k)Q_1\chi(k) + u_*^T(k)Q_c u_*(k)]\right\} \tag{24}$$

where $\chi(k) = X(k) - X_d(k)$, Q_0 and Q_1 are $n \times n$ nonnegative matrices, and Q_c is a positive scalar. $X(k)$ is the practical value of the system state, and $X_d(k)$ is the designed working temperature. The physical descriptions of the terms in the quadratic performance index are as follows. The state variable error at the final time is represented by $\chi^T(k_f)Q_0\chi(k_f)$, and the other two terms specify the requirements for the state variable error and the control variable in the overall process, which includes the error $\chi^T(k)Q_1\chi(k)$ and the energy $u^T(k)Q_c u(k)$. The random optimal control problem in Eq. (21a) is used to obtain the array $u(0), u(1), u(2)\cdots u(k_f-1)$ that minimizes the index function $J_i(u_*)$. The error $\chi^T(k)Q_1\chi(k)$ and the energy $u^T(k)Q_2u(k)$ are interdependent. Reducing the state variable error requires magnification of the control effort, which might not be obtained in practice when the state variable error is too large. On the other hand, to reduce the control effort, the request for the reduction of the state variable error will need to be compromised in a way. In this work, we set $Q_0 = Q_1 = Q_s \times I$. Q_s and Q_c are defined as follows:

$$Q_s = \begin{bmatrix} \frac{1}{\varepsilon_{1,\max}} & 0 & 0 & 0 & 0 \\ 0 & \ddots & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{\varepsilon_{i,\max}} & 0 & 0 \\ 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{\varepsilon_{N,\max}} \end{bmatrix} \tag{25}$$

$$Q_c = \frac{1}{u_{\max}} \tag{26}$$

In the preceding equations, $\varepsilon_{i,\max}$ is the maximum tolerance of the ith state variable error, and $u_{\max}$ is the maximum control variable. In this work, the value of $\varepsilon_{i,\max}$ depends on the temperature error, and $u_{\max}$ is an adjustable parameter with its maximum value equal to the maximum heat dissipated by the chosen cooling system.

The LQG regulator is used to optimize the feedback control vector $u_*(k)$. Following rearrangement of the items in parentheses in Eq. (22), we get

$$\hat{u}(k) = -K_r(k)\chi(k) - (\Lambda^T\Lambda)^{-1}\Lambda^T\Gamma\hat{q}(k) \tag{27}$$

In Eq. (27), the regular gain $K_r(k)$ is given by

$$K_r(k) = [\Lambda^T P_1(k+1)\Lambda + Q_c]^{-1}\Lambda^T P_1(k+1)\Phi \tag{28}$$

where P_1 is the positive semidefinite stabilizing solution of the discrete-time Riccati equation is shown as [17]

$$P_1(k) = \Phi^T\{P_1(k+1) - P_1(k+1)\Lambda[\Lambda^T P_1(k+1)\Lambda + Q_c]^{-1}\Lambda^T P_1(k+1)\}\Phi + Q_1, \qquad k < k_f \tag{29}$$

with boundary condition $P_1(k_f) = Q_0$, $k_f = (t_f/\Delta t) + 1$.

Based on Eq. (29), by inversely calculating from $k = k_f$ to $k = 1$, $P_1(k)$ can be obtained. In Eq. (27), $\chi(k)$ denotes the state variable error between the design working temperature and the practical value of the system state. To simulate measurement temperature of the thermal conductor of the package seal for the integrated circuit under heating $q_{\text{exact}}(k)$ and the amount of heat which must be dissipated $\hat{u}(t)$, the system response equation is as follows:

$$X(k+1) = \Phi X(k) + \Lambda\hat{u}(k) + \Gamma q_{\text{exact}}(k) \tag{30}$$

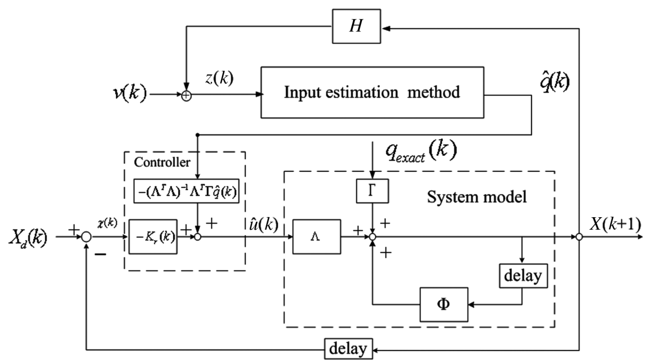

Fig. 3 LQG/IE control flowchart.

To verify the performance of the controller, the root mean square error (RMSE) is defined:

$$\mathrm{RMSE} = \left[\frac{1}{k_f}\sum_{k=1}^{k_f}[X_d(k) - X(k)]^2\right]^{\frac{1}{2}} \tag{31}$$

Figure 3 presents a detailed control flowchart of the LQG/IE algorithm. The following section describes the numerical calculation procedure.

Numerical Calculation Procedure

Unlike the unknown heat flux generated during electronic device operation at $x = 0$, on the package seal, an intensive convection (or natural convection) boundary condition is established by connecting the device to heat-dissipating equipment on the opposite side of the package seal. In the process of estimation, using the LQG/IE algorithm, the initial values of the parameters must be given. The overall calculation procedure is as follows.

1) The discrete-time state equation, Eq. (3), and the measurement equation, Eq. (8), are established.

2) Set the weighting matrices in the quadratic performance index $J(u_*)$ as follows. Compute Q_s from Eq. (25) while assuming that $\varepsilon_{i,\max} = 1°\mathrm{C}$, $i = 1, 2, \ldots, N$. In this work, we set $Q_0 = Q_1$. Q_c is obtained by Eq. (26) after setting the appropriate maximum control variable $u_{\max}$.

3) Substitute $P_1(k)$, which is computed by solving the Riccati equation, Eq. (29), in the backward direction starting with the terminal condition $P_1(k_f) = Q_0$, into Eq. (28), to obtain the regular gain $K_r(k)$.

4) The initial conditions of T_0, $X_d(k)$, $\hat{q}(-1)$, $P_b(-1)$, and $M(-1)$ are assumed.

5) The framework of the IE method, given by Eqs. (9–20), is used to estimate the heat-flux input $\hat{q}(k)$.

6) Compute the error $\chi(k)$ between $X(k)$ and $X_d(k)$. If the error is negative, which means that the simulated measurement temperature $X(k)$ is less than the designed working temperature, then substitute the natural convection coefficient h_{air} into Eq. (1c) to obtain $\hat{u}(k)$; otherwise, compute $\hat{u}(k)$, using Eq. (27).

7) Substitute $\hat{q}(k)$ and $\hat{u}(k)$, which are computed in steps 5 and 6, respectively, into the system response equation, Eq. (30), to compute $X(k+1)$.

8) Repeat steps 5–7 until the final time (t_f) is reached.

Results and Discussion

Three time-varying heat-flux models are examined experimentally to verify the effectiveness and the precision of the proposed LQG/IE algorithm, applied in heat-flux estimation and optimal electronic package heat-dissipation control. The following physical quantities were used in the calculation:

$$\rho = 1856\ \mathrm{kg/m^3}, \quad C_p = 1078\ \mathrm{J/(kg°C)} \quad K_s = 0.74\ \mathrm{W/(m°C)}$$

The total time t_f is assumed to be 120 s, and the sampling interval is $\Delta t = 0.01$ s. The thickness of the package material is $L = 2.0 \times 10^{-4}$ m. The thermocouple is placed on top of the package material at $x = L$, and the total number of spatial nodes is $N = 10$. The initial temperature is $T_0 = 20°\mathrm{C}$, and the designed working temperature is $X_d = 40°\mathrm{C}$.

The initial conditions for the IE method were given by $\bar{X}(-1/-1) = \begin{bmatrix} T_0 & T_0 & \cdots & T_0 \end{bmatrix}^T$, $Q = 10^3$, and $R = \sigma^2$, where σ denotes the standard deviation of the measurement. The state filtering-error-covariance matrix for the Kalman filter is

$$P(-1/-1) = \begin{bmatrix} 10^8 & & & & 0 \\ & 10^8 & & & \\ & & \ddots & & \\ & & & 10^8 & \\ 0 & & & & 10^8 \end{bmatrix} \tag{32}$$

The recursive least-squares algorithm initial conditions were given by $\hat{q}(-1) = 0$, $\gamma = 0.875$, $P_b(-1) = 10^8$, and $M(-1) = 0$. Of them, the initial values of $P(-1/-1)$ and $P_b(-1)$ are normally unknown, and the initial error is assumed to be large, which causes the estimator to converge rapidly within a short transition period and to steadily track the time-varying coefficients.

Based on a guessed value for $q_{\mathrm{exact}}(t)$ of the heat flux generated by the electronic device and a computation of the amount of heat which must be dissipated $\hat{u}(t)$, using the LQG regulator, the distribution of the temperature field can be determined using the direct heat conduction method from Eq. (28). Furthermore, the measurement error can be added to the determined temperature field to yield the measurement temperature:

$$z(t) = T(x_m, t) + \eta\sigma \tag{33}$$

where η is a random variable in the range $-2.576 \le \eta \le 2.576$ and σ denotes the standard deviation of the measurements. By assuming there is a thermocouple embedded on the top of the package seal at $x = L$, the simulated measurement temperature can be determined. The estimation and optimal thermal control are implemented by applying three time-varying heat-flux models, as in cases 1–3. The results are compared with those obtained using the LQG regulator.

Case 1: Estimation using a square heat-flux wave.

Suppose that the time-varying heat-flux input is a square wave:

$$q(t) = \begin{cases} 0 & 0 \le t < 20, 40 < t \le t_f \\ 4000 & 20 \le t \le 40 \end{cases} \ (\mathrm{W/m^2}) \tag{34}$$

Two covariances of measurement noise, $R = 10^{-4}$ and $R = 10^{-6}$, and weights of the control $Q_c = 10^{-3}$ and $Q_c = 10^{-2}$, are considered. Figure 4 presents the inverse estimation of the square heat-flux wave using the IE mthod, indicating that the IE mthod can

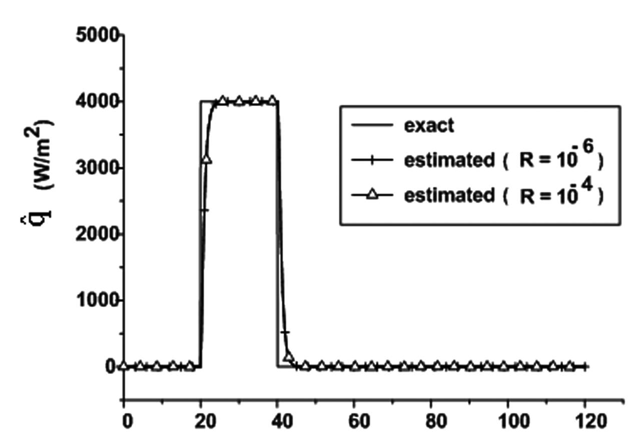

Fig. 4 Estimated heat flux using IE algorithm with varying *R* for case 1.

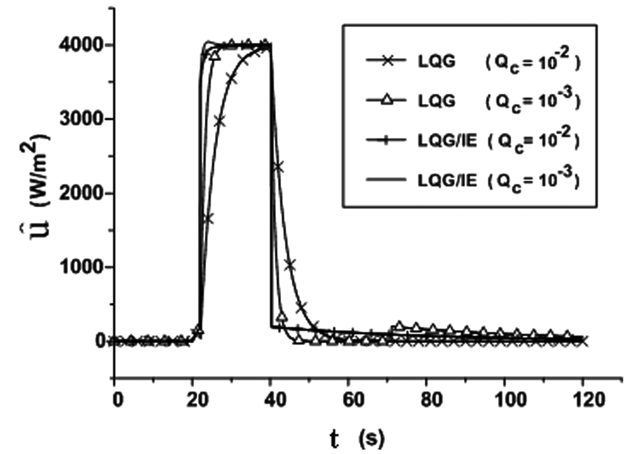

Fig. 5 Dissipative heat flux for various control algorithms for case 1 ($R = 10^{-6}$).

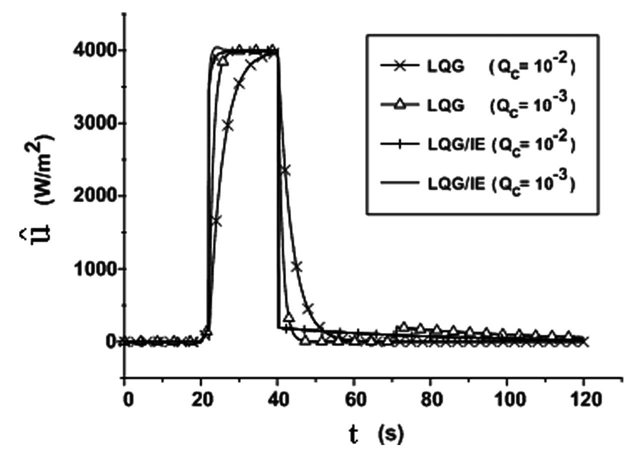

Fig. 6 Dissipative heat flux for various control algorithms for case 1 ($R = 10^{-4}$).

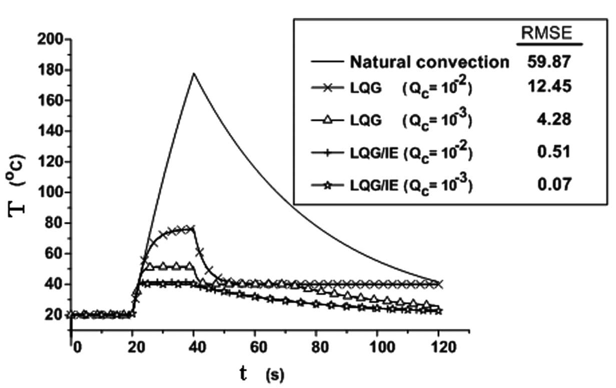

Fig. 7 Comparison of surface temperature predictions for various control algorithms for case 1 ($R = 10^{-6}$).

effectively estimate the unknown time-varying heat flux in real time, based on simulated temperature measurement. Figures 5 and 6 present the optimal control heat-flux results, revealing that the LQG/IE algorithm can provide a short time response and more precise tracking than that obtained using only the LQG regulator.

The LQG/IE algorithm is less sensitive to the weights of the control. As shown in Figs. 7 and 8, the LQG regulator reacts more slowly as Q_c increases and larger errors in the simulated temperature measurement are generated. Figure 7 shows that when $R = 10^{-6}$, the RMSE values based on the LQG regulator are 4.28 and 12.45, and based on the LQG/IE method, the values are 0.07 and 0.51 for

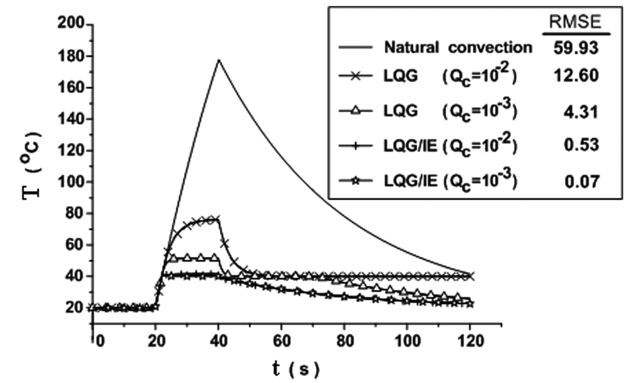

Fig. 8 Comparison of surface temperature predictions for various control algorithms for case 1 ($R = 10^{-4}$).

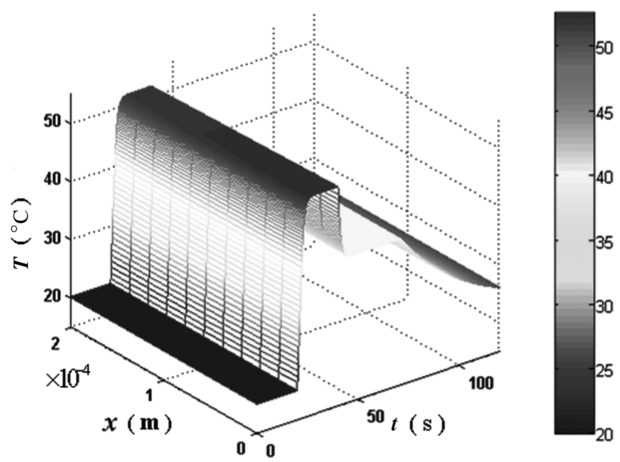

Fig. 9 Temperature profile using LQG algorithm for case 1 ($R = 10^{-4}$).

$Q_c = 10^{-3}$ and $Q_c = 10^{-2}$, respectively. Figure 8 shows that, when $R = 10^{-4}$, the RMSE values based on the LQG regulator are 4.30 and 12.60, and based on the LQG/IE method are 0.07 and 0.52 for $Q_c = 10^{-3}$ and $Q_c = 10^{-2}$, respectively. This error arises from the fact that the LQG regulator cannot recognize when an unknown thermal input acts on the thermal conductor. The IE method, however, can estimate an unknown thermal input in real time. Combining the IE method and the LQG regulator, which computes the optimal thermal control heat flux, can yield rapid and precise heat-dissipation results. The temperature field distributions obtained using the LQG/IE algorithm and the LQG regulator shown in Figs. 9 and 10 are consistent with the results in Figs. 7 and 8. Therefore, the LQG/IE algorithm can maintain a universal system temperature within an ideal range and is suited to heat-dissipation applications in unknown systems with time-varying or transient thermal input sources.

Case 2: Estimation using a compound square heat-flux wave.

Suppose that the time-varying heat-flux input is

$$q(t) = \begin{cases} 0 & 0 \le t \le 10, 110 \le t \le t_f \\ 2000 & 10 < t < 30 \\ 3000 & 30 \le t < 70 \\ 3500 & 70 \le t < 90 \\ 4500 & 90 \le t < 110 \end{cases} \quad (\mathrm{W/m^2}) \qquad (35)$$

Two covariances of measurement noise, $R = 10^{-4}$ and $R = 10^{-6}$, are considered. Figure 11 depicts inverse estimation of the square heat-flux wave using the IE method and verifies that the IE method with the simulated temperature can effectively estimate the unknown time-varying heat flux in real time. The covariance of measurement noise is set to $R = 10^{-4}$, and the weights of the control are set to $Q_c = 10^{-2}$ and $Q_c = 10^{-3}$. Figure 12 presents the optimal control

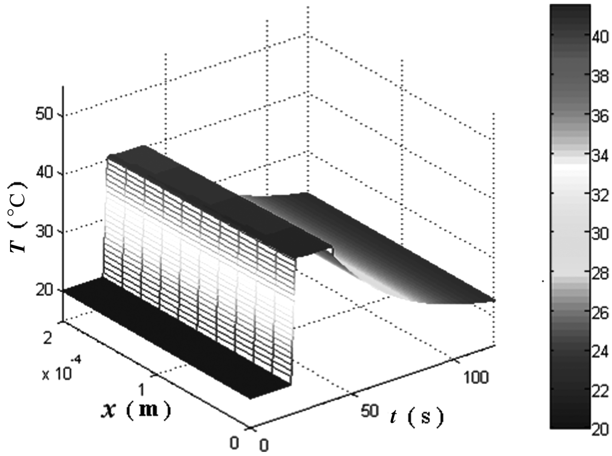

Fig. 10 Temperature profile using LQG/IE algorithm for case 1 ($R = 10^{-4}$).

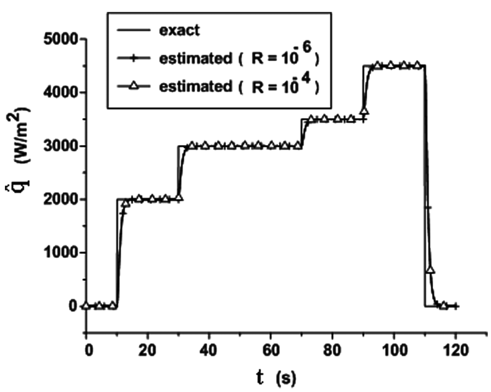

Fig. 11 Estimated heat flux using IE algorithm with varying R for case 2.

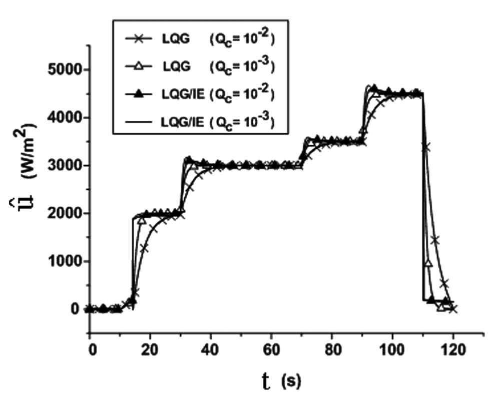

Fig. 12 Dissipative heat flux for various control algorithms for case 2 ($R = 10^{-4}$).

heat-flux results. Figure 13 plots the optimal thermal control surface temperature results, and Fig. 14 presents the temperature profile using the LQG/IE algorithm. All of these results are consistent with the discussion in case 1. Figure 13 shows that the RMSE values based on the LQG regulator are 8.51 and 26.55, and the values based on the LQG/IE method are 0.25 and 1.35 for $Q_c = 10^{-3}$ and $Q_c = 10^{-2}$, respectively. The LQG/IE algorithm yields a shorter response time and more precise outcome than the LQG regulator. It is also less sensitive to the weights of the control and offers better electronic package heat dissipation.

Case 3: Estimation using a compound square impulse heat-flux wave.

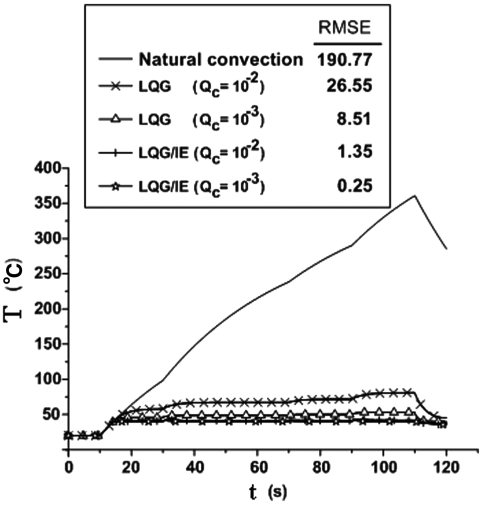

Fig. 13 Comparison of surface temperature predictions for various control algorithms for case 2 ($R = 10^{-4}$).

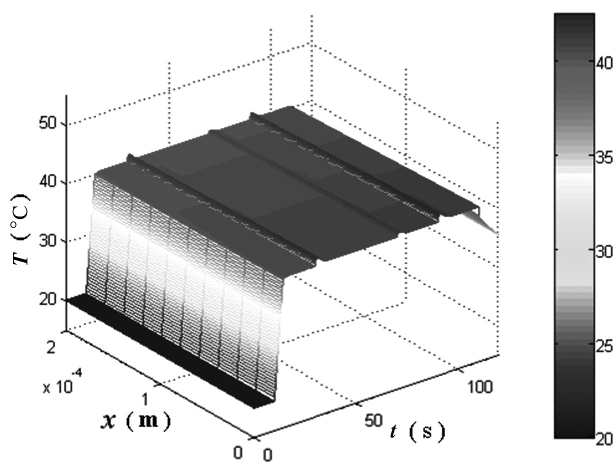

Fig. 14 Temperature profile using LQG/IE algorithm for case 2 ($R = 10^{-4}$).

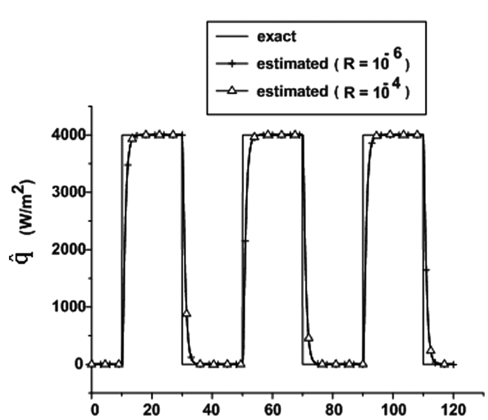

Fig. 15 Estimated heat flux using IE algorithm with varying R for case 3.

Suppose that the time-varying heat-flux input is

$$q(t) = \begin{cases} 0 & 0 \le t < 10, 30 < t < 50, 70 < t < 90, 110 < t \le t_f \\ 4000 & 10 \le t \le 30, 50 \le t \le 70, 90 \le t \le 110 \end{cases} (\mathrm{W/m^2}) \quad (36)$$

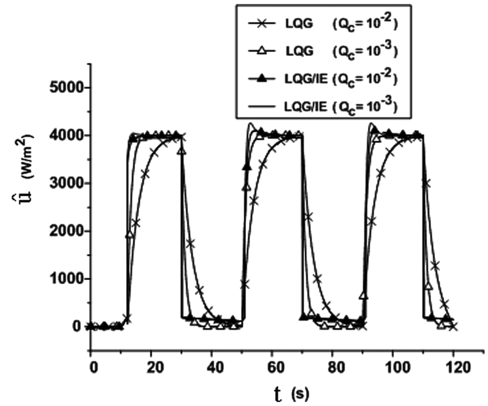

Fig. 16 Dissipative heat flux for various control algorithms for case 3 ($R = 10^{-4}$).

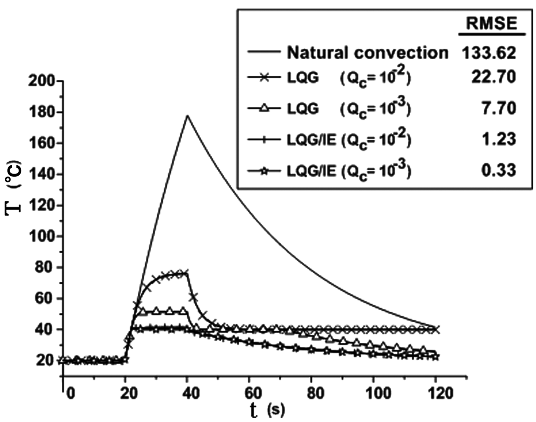

Fig. 17 Comparison of surface temperature predictions for various control algorithms for case 3 ($R = 10^{-4}$).

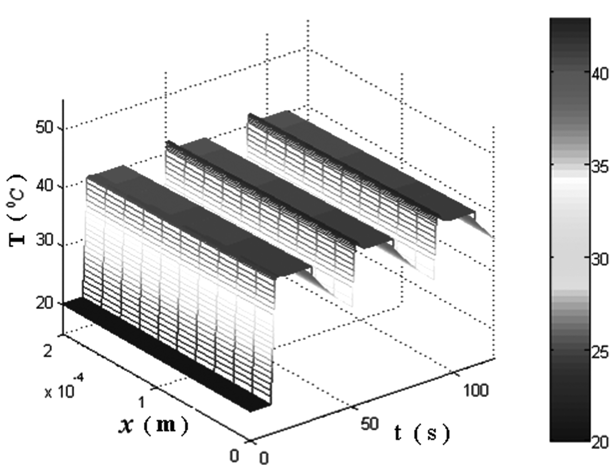

Fig. 18 Temperature profile using LQG/IE algorithm for case 3 ($R = 10^{-4}$).

Given two covariances of measurement noise, $R = 10^{-4}$ and $R = 10^{-6}$, the inverse estimation of the square heat-flux wave using the IE method is shown in Fig. 15, which demonstrates that the IE method effectively estimates the unknown time-varying heat flux in real time. Suppose that the standard deviation of the measurement noise is $\sigma = 10^{-2}$, and the weights of the control are $Q_c = 10^{-2}$ and $Q_c = 10^{-3}$. Figure 16 plots the optimal control heat-flux results. Figure 17 plots the optimal thermal control surface temperature results, and Fig. 18 plots the comparison results between the temperature field distributions. All of these results are consistent with those in cases 1 and 2. Figure 17 shows that the RMSE values based on the LQG regulator are 7.70 and 22.70, and the values based on the LQG/IE method are 0.33 and 1.23 for $Q_c = 10^{-3}$ and $Q_c = 10^{-2}$, respectively. The LQG/IE algorithm provides a quicker and more precise outcome than the LQG regulator alone, and is less affected by the weights of the control. It quickly yields precise heat-dissipation results. The approach described herein will ensure the device life span, enhance the system reliability, and solve the heat-dissipation problem associated with electronic device development for higher power capacity, and yield greater effectiveness and lower weight.

Conclusions

This work developed a more effective LQG/IE algorithm to resolve thermal control issues. The IE method inversely estimates in real time the heat flux generated by electronic devices based on a temperature simulated from the thermocouple measurements. The LQG regulator is adopted to analyze the feedback gain to control the heat dissipation. Three simulated time-varying heat fluxes were used to verify LQG/IE algorithm performance. The results in the three cases demonstrated that the LQG/IE algorithm outperforms the LQG regulator with a shorter response time and more accurate heat-flux tracking. Therefore, optimal heat-dissipation control using an LQG/IE algorithm can provide temperature monitoring protection for electronic devices in an operational environment. Further work will involve implementing the LQG/IE algorithm in electronic devices.

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